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Mann–Whitney U-test Wilcoxon Test

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Definition of Mann–Whitney U test

- The Mann-Whitney U test is used to compare differences between two independent groups when the dependent variable is either ordinal or continuous, but not normally distributed.

Unpaired t-test = Mann-Whitney U test

- This test can be used to investigate whether two independent samples were selected from populations having the same distribution. A similar nonparametric test used on dependent samples is the Wilcoxon signed-rank test.

Assumptions

- dependent variable should be measured at the ordinal or continuous level.
 - independent variable should consist of two categorical, independent groups.
 - independence of observations.
 - two variables are not normally distributed.
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- ***H0: The two populations are equal***
 - ***H1: The two populations are not equal***

Calculation

$$U_1 = n_1 * n_2 + \frac{n_1 * (n_1 + 1)}{2} - R_1$$

$$U_2 = n_1 * n_2 + \frac{n_2 * (n_2 + 1)}{2} - R_2$$

where,

- $n_{1,2}$ - the sample size of groups
- $R_{1,2}$ - the sum of the ranks for groups

$$U_1 + U_2 = n_1 n_2$$

The test statistic for the Mann Whitney U Test is denoted **U** and is the smaller of U1 and U2.

Results

$U_{obs} \leq U_{crit}$ reject H_0

$U_{obs} > U_{crit}$ do not reject H_0

Critical Values of the Mann-Whitney U
(Two-Tailed Testing)

n_2	α	n_1																		
		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
3	.05	--	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	
	.01	--	0	0	0	0	0	0	0	0	1	1	1	2	2	2	2	3	3	
4	.05	--	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14	
	.01	--	--	0	0	0	1	1	2	2	3	3	4	5	5	6	6	7	8	
5	.05	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20	
	.01	--	--	0	1	1	2	3	4	5	6	7	7	8	9	10	11	12	13	
6	.05	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27	
	.01	--	0	1	2	3	4	5	6	7	9	10	11	12	13	15	16	17	18	
7	.05	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	
	.01	--	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	24	
8	.05	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41	
	.01	--	1	2	4	6	7	9	11	13	15	17	18	20	22	24	26	28	30	
9	.05	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48	
	.01	0	1	3	5	7	9	11	13	16	18	20	22	24	27	29	31	33	36	

Example

- Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. A total of $n=10$ participants are randomized to receive either the new drug or a placebo. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of the assigned treatment. The data are shown below.
- Is there a difference in the number of episodes of shortness of breath over a 1 week period in participants receiving the new drug as compared to those receiving the placebo? By inspection, it appears that participants receiving the placebo have more episodes of shortness of breath, but is this statistically significant?

Placebo	7	5	6	4	12
New Drug	3	6	4	2	1

Wilcoxon signed-rank test

The **Wilcoxon signed-rank test** is a non-parametric statistical hypothesis test used to compare two related samples, matched samples, or repeated measurements on a single sample to assess whether their population mean ranks differ (i.e. it is a **paired difference test**).

It can be used as an alternative to the paired Student's t-test (also known as "t-test for matched pairs" or "t-test for dependent samples") when the distribution of the difference between two samples' means cannot be assumed to be normally distributed

Calculation

	A	B	C	D	E	F	G	H	I	J	K
1	Wilcoxon Signed-Rank Test for Paired Samples										
2											
3	Person	Right	Left	Diff	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks			
4	1	50	47	3	3	4.5	4.5			α	0.05
5	2	45	45	0						tails	2
6	3	33	31	2	2	2.5	2.5			n	14
7	4	22	24	-2	2	2.5		2.5		T	35.5
8	5	99	78	21	21	14	14			T-crit	21
9	6	79	76	3	3	4.5	4.5			sig	no
10	7	4	13	-9	9	11		11			
11	8	36	46	-10	10	12		12			
12	9	62	45	17	17	13	13				
13	10	51	44	7	7	9	9				
14	11	27	23	4	4	6.5	6.5				
15	12	15	14	1	1	1	1				
16	13	26	34	-8	8	10		10			
17	14	83	79	4	4	6.5	6.5				
18	15	86	81	5	5	8	8				
19	T						69.5	35.5			

Results

$W_{\text{test}} \leq W_{\text{crit}}$ reject H0

$W_{\text{test}} > W_{\text{crit}}$ do not reject H0

n	alpha values						
	0.001	0.005	0.01	0.025	0.05	0.10	0.20
5	--	--	--	--	--	0	2
6	--	--	--	--	0	2	3
7	--	--	--	0	2	3	5
8	--	--	0	2	3	5	8
9	--	0	1	3	5	8	10
10	--	1	3	5	8	10	14
11	0	3	5	8	10	13	17
12	1	5	7	10	13	17	21
13	2	7	9	13	17	21	26
14	4	9	12	17	21	25	31
15	6	12	15	20	25	30	36
16	8	15	19	25	29	35	42
17	11	19	23	29	34	41	48

Thank you for your attention!!!